Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

ME - SEMESTER–1 • EXAMINATION – WINTER 2014

Subject Code: 2714303 **Date:09/01/ 2015**

Subject Name: Theory of Elasticity & Plasticity

Time: 2:30 to 5:00 Pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks**

Q.1 **(a)** Derive the basic differential equation for equilibrium in Cartesian co-ordinate system. **07**

(b) If the components of the stress tensor ů in MPa, at some point P in a beam with respect to (x,y,z) co-ordinates is given as below: **07**

$$\sigma = \begin{matrix} 35 & -25 & 0 \\ -25 & -15 & 0 \\ 0 & 0 & 0 \end{matrix}$$

(a) Find the stress vector t_n on the plane whose normal is given by nx=ny=nz = $1/\sqrt{3}$

(b) Find out the normal ($\mathring{u}_n$) and shear ($\acute{U}_n$)components of t_n.

Q.2 **(a)** Commencing from the equations defining the state of stress at a point derive the general stress relationship for the normal stress on an inclined plane. **07**

$$\sigma_n = \sigma_{xx} l^2 + \sigma_{zz} n^2 + \sigma_{yy} m^2 + 2\sigma_{xy} lm + 2\sigma_{yz} mn + 2\sigma_{zx} ln$$

Show that the relationship reduces for the plane stress system to well-known equation:

$$\sigma_n = \tfrac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \tfrac{1}{2}(\sigma_{xx} - \sigma_{yy})\cos 2\theta + \sigma_{xy} \sin 2\theta$$

(b) Explain Compatibility of strain in detail. Derive the compatibility relations of strain for a 2-D system. **07**

OR

(b) Draw a neat sketch of the displacement of a cubic element and hence derive the equations of linear and shear strains in Cartesian co-ordinate system. **07**

Q.3 **(a)** Derive Airyô stress function in Polar co-ordinate system for an axi-symmetric stress distribution. **07**

(b) The stress tensor at a point is given as $\begin{bmatrix} 200 & 160 & -120 \\ 160 & -240 & 100 \\ -120 & 100 & 160 \end{bmatrix}$ kN/m^2 **07**

Determine the strain tensor at this point. Take E= 210 x 10^6 kN/m^2 and μ=0.3

OR

Q.3 **(a)** For the curved beam subjected to moment: M = 150 kJ, internal & external radii: a = 150 mm & b = 350 mm respectively, calculate radial and transverse stresses at inner, outer and every quarter thickness points and plot their variations using the following equations with usual notations: **07**

Radial stress:

$\sigma_r = ï\ {}^{4M}/_{N}\ [a^2b^2/r^2 \ln (b/a) + b^2 \ln (r/b) + a^2 \ln (a/r)]$

Tangential stress:

$\sigma_\theta = ï\ {}^{4M}/_{N}\ [ï\ a^2b^2/r^2 \ln (b/a) + b^2 \ln (r/b) + a^2 \ln (a/r) + b^2\ ï\ a^2]$

Here; $N = (b^2\ ï\ a^2)^2\ ï\ 4\ a^2b^2\ [\ln (b/a)]^2$

(b) Using Swift construction, find normal and resultant shear stress on a plane whose normal has directions cosines are l = 0.342, m = 0.405 respectively w.r.t. Principal stresses: P_1 = 900 MPa (Tensile), P_2 = 200 MPa (Tensile) and P_3 = 300 MPa (Compressive). **07**

Q.4 **(a)** State the generalized Hookeôs law. Write down the constitutive relations for an Isotropic material in 2D. **10**

Calculate the volumetric strain for the following data: $\mathring{u}_x$ =200 N/mm^2, $\mathring{u}_y$ =150 N/mm^2, $\mathring{u}_z$ =120 N/mm^2, E = 210 kN/mm^2, μ = 0.3.

(b) State the various failure criterion that govern the failure of materials under static loading. Also give the statement of Tresca and Von Mises yield criterion. **04**

OR

Q.4 **(a)** If the stress tensor for a three dimensional stress system is given as below and one of the principal stress has a value of 40 MN/m^2, determine the values of the three eigen vectors. **07**

$$\begin{matrix} 30 & 10 & 10 \\ 10 & 0 & 20 \\ 10 & 20 & 0 \end{matrix}$$

Q.4 **(b)** Prove the following failure criteria according to the Distortion Energy theory: **07**

$$2\sigma_y^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

Q.5 **(a)** Explain the concept of stability of structures. Give the basis of stability of analysis for a slender straight column as well as for a column initially bent. **07**

(b) Draw the stress distribution diagram for thick wall cylinder subjected to only internal pressure. Also explain the boundary conditions utilized in TOE for **07**

(i) Thick wall cylinder subjected to only external pressure

(ii) Plate element with circular hole of large radius

OR

Q.5 **(a)** Draw a neat sketch of an element subjected to body forces, radial stresses, transverse stresses and shear stresses, derive the basic differential equations of equilibrium in 2-D Polar co-ordinate system. **07**

(b) Write Short notes on any two: **07**

(i) Parameters that govern the theories of failure

(ii) Assumptions made in Linear / Classical theory of elasticity

(iii) Hydrostatic and Deviatoric stresses

(iv) Ductile failure vs Brittle failure