Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M. E. - SEMESTER – III • EXAMINATION – WINTER • 2014

Subject code: 730404 **Date: 27-11-2014**

Subject Name: Applied Linear Algebra in Engineering

Time: 02:30 pm - 05:00 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Prove that the set of all pairs of real numbers of the form (1, x) with the operations $(1, y) + (1, y') = (1, y + y')$ and $k(1, y) = (1, ky)$ is a vector space. **07**

(b) Find the Bases for row space, column space and null space for **07**

$$A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$

Q.2 **(a)** Solve the following system of equation by Gaussian elimination method. **07**
$3x + 2y - z = -15$, $5x + 3y + 2z = 0$, $3x + y + 3z = 11$, $-6x - 4y + 2z = 30$.

(b) Solve the following system of equation by Gauss-Jordan elimination. **07**
$10y - 4z + w = 1$, $x + 4y - z + w = 2$, $3x + 2y + z + 2w = 5$,
$-2x - 8y + 2z - 2w = -4$, $x - 6y + 3z = 1$.

OR

(b) Write reduced row echelon form for any 3 x 3 invertible matrix. Find inverse of the matrix A by Gauss – Jordan method for $A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ **07**

Q.3 **(a)** Define inner product space. Find <p, q>, || p ||, || q || for respective inner product spaces with usual operations. **07**
(1) $p = 1 + x - 2x^2$, $q = 3 - 4x + x^2$
(2) $p = (-2, 3, -4)$, $q = (1, -4, 2)$

(b) Define subspace of a vector space. Check whether the following sets are subspaces or not. **07**

(1) All matrices of the form $\begin{bmatrix} a & a \\ -a & -a \end{bmatrix}$, where $a \in R$.

(2) All matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d \in R$, $a + b + c + d = 0$.

OR

Q.3 **(a)** Define orthogonal vectors and orthonormal vectors. Apply Gram – Schmidt process of orthogonalization to find orthonormal vectors for **07**
$x_1 = (3, 0, 4)$, $x_2 = (-1, 0, 7)$ and $x_3 = (2, 9, 11)$.

(b) Find the angle between u and v for **07**
(1) $u = 3 - x + 4x^2$, $v = 1 + 2x - 4x^2$
(2) $u = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$, $v = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$

Q.4 **(a)** Verify Cayley – Hamilton's theorem for $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$. **07**

(b) Find A^5 and A^{-1} using Cayley – Hamilton's theorem for $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 3 \\ 2 & -2 & 1 \end{bmatrix}$. **07**

OR

Q.4 **(a)** Check whether following are linear transformation or not **07**
(1) F: $M_{mn} \rightarrow M_{nm}$ where F (A) = A^T.
(2) T: F $(-\infty, \infty) \rightarrow$ F $(-\infty, \infty)$ where T (f(x)) = 1 + f(x).

(b) Let T: $R^2 \rightarrow R^3$ be defined by T (x, y) = (x + 2y, – x, 0). Find the matrix $[T]_{B',B}$ with respect to the bases B = $\{u_1, u_2\}$ and B' = $\{v_1, v_2, v_3\}$ where $u_1 = (1, 3)$, $u_2 = (-2, 4)$, and $v_1 = (1, 1, 1)$, $v_2 = (2, 2, 0)$, $v_3 = (3, 0, 0)$. **07**

Q.5 **(a)** Solve the following system of equation using Gauss – Seidel Method. **07**
x + y + 54z = 110, 27x + 6y – z = 85, 6x + 15y + 2z = 72.

(b) Find the least squares solution of the linear system A X = B, and find the orthogonal projection of B onto the column space of A. Where A = $\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}$. **07**

OR

Q.5 **(a)** Solve the following system of equation using Jacobi's method. **07**
10x + y + z = 12, 2x + 10y + z = 13, 2x + 2y + 10z = 14.

(b) Find the orthogonal projection of u onto the subspace of R^3 spanned by the vectors v_1 and v_2, where u = (2, 1, 3), $v_1 = (1, 1, 0)$, $v_2 = (1, 2, 1)$. **07**

****