Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE / ME / MBA / MCA - SEMESTER– • EXAMINATION – WINTER 2014

Subject Code: 2710710 **Date:06/01/ 2015**

Subject Name: LINEAR ALGEBRA

Time: **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Are the vectors $\alpha_1 = (1,1,2,4)$, $\alpha_2 = (2,-1,-5,2)$, $\alpha_3 = (1,-1,-4,0)$, $\alpha_4 = (2,1,1,6)$ linearly independent in R^4? **03**

(b) Check whether the following set of vectors form a subspace. **04**

(i) $S = \{\alpha / \alpha = (a_1, a_2, ...a_n);\ a_1 a_2 = 0;\ a_i \in R, \forall i\}$

(ii) Set of all functions of the form $k_1 + k_2 \sin x$, in $F(-\infty, \infty)$.

(c) Let V be a vector space over the field F. Show that the intersection of two subspaces of V is a subspace of V. What can you say for the union of two subspaces of V? **07**

Q.2 **(a)** Let V and W be two vector spaces over the field F. Let T and U be linear transformations from V into W. Then prove that $T+U$ and TU are also linear transformations. **07**

(b) If T is linear transformation defined by **07**

$$T(1,0,0) = (0,1,0,2)$$
$$T(0,1,0) = (0,1,1,0)$$
$$T(0,0,1) = (0,1,-1,4)$$

Then find range of T, null space of T and dimensions of $R(T)$ and $N(T)$.

OR

(b) Let T be the linear operator on R^3 defined by **07**

$T(x_1, x_2, x_3) = (3x_1,\ x_1 - x_2,\ 2x_1 + x_2 + x_3)$.

Is T invertible ? If so, find a rule for T^{-1}.

Q.3 **(a)** Let C be the field of complex numbers in algebra of polynomials and let $f = x^2 + 2$, then **07**

1. If $z \in C$; where $z = \dfrac{1+i}{1-i}$, then show that $f(z) = 1$.
2. Find $f(B)$, where $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$.
3. If the linear operator T defined by $T(c_1, c_2, c_3) = (i\sqrt{2}\ c_1, c_2, i\sqrt{2}\ c_3)$, then find $f(T)$.
4. If $g = x^4 + 3i$ then find $f(g)$.

(b) State and prove Cayley-Hamilton theorem. **07**

Verify this result for $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

OR

Q.3 **(a)** Determine the eigen values and the corresponding eigen spaces for the matrix $A=\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ **07**

(b) Let S_1, S_2 & S_3 be the subspaces of R^3 generated by **07**
$S_1=\{(a,b,c)/a+b-2c=0\}$, $S_2=\{(a,b,c)/a=b\}$ &
$S_3=\{(a,b,c)/a+2c=0, b-4c=0\}$.
Determine (a) a basis for $S_1 \cap S_2$ and (b) the dimension of S_1+S_3.

Q.4 **(a)** Define the following terms (i) T - invariant subspace **07**
(ii) Restriction of a linear transformation
Let T be a linear transformation on R^3 defined by
$T(x,y,z)=(-3x+3y-2z,\ -7x+6y-3z,\ x-y-2z)$.
(i) Show that the subspace $S=\left\{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\right\}$ is T - invariant.
(ii) Determine the matrix of T restricted to S with respect to the basis vectors of S.

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1,u_2,u_3\}$ into an orthonormal basis where **07**
$u_1=(0,1,2)$, $u_2=(-1,0,1)$, $u_3=(-1,1,3)$.

OR

Q.4 **(a)** Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix $A=\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Find the minimal polynomial for T. **07**

(b) Verify that the matrix $A=\begin{bmatrix} \frac{1+i}{2} & \frac{1+i}{2} \\ \frac{1-i}{2} & \frac{-1+i}{2} \end{bmatrix}$ is unitary. Also find its inverse. **07**

Q.5 **(a)** Show that $\langle u,v\rangle=\frac{1}{4}u_1v_1+\frac{1}{16}u_2v_2$ forms an inner product space on R^2. Sketch the unit circle in an xy-coordinate system in R^2. **07**

(b) Solve the following linear system using Doolittleôs method- **07**

$$\begin{aligned} 3x+5y+2z &= 8 \\ 8y+2z &= -7 \\ 6x+2y+8z &= 26 \end{aligned}$$

OR

Q.5 **(a)** Let a linear transformation on C^3 be defined by $Lx=(x_1+i\,x_2, x_2+i\,x_3, x_3+i\,x_1)$ where $x=(x_1,x_2,x_3)$ is in C^3. Determine the adjoint of L. Show that L^* is transposed conjugate of L. **07**

(b) Apply the power method to the following symmetric matrix to obtain the **07**

dominant eigen value by taking the initial approximation as $x_0 = [1 \quad 1 \quad 1]^T$ -

$$A = \begin{bmatrix} 0.49 & 0.02 & 0.22 \\ 0.02 & 0.28 & 0.20 \\ 0.22 & 0.20 & .040 \end{bmatrix}$$

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