Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC 2ND Semester Examination – July- 2011

Subject code:X20001

Subject Name: MATHEMATICS-II

Date:11/07/2011 **Time: 10:30 am – 01:30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Attempt the following.

a) Prove that $\frac{\beta(m+1,n)}{m}=\frac{\beta(m,n+1)}{n}=\frac{\beta(m,n)}{m+n}$. **3**

b) Express the integral $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of Gamma functions. **3**

c) Define Beta function and compute $\beta(2.5,1.5)$. **2**

d) Find : $L\{(t+2)^2 e^t\}$. **2**

e) Solve : $\frac{d^2y}{dx^2}+a^2y=0$. **2**

f) Solve : $\frac{\partial^2 z}{\partial x^2}=xy$. **2**

Q.2 **(a)** (1) Find the Laplace Transformations of **4**

a) $\frac{1-e^t}{t}$

b) $te^{-2t}\sin 2t$

(2) Using Convolution theorem evaluate $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$. **3**

(b) (1) Find the inverse Laplace Transforms of **4**

a) $\frac{3s}{s^2+2s-8}$

b) $\log\left(\frac{s+a}{s+b}\right)$

(2) Evaluate : $\int_0^\infty te^{-2t}\sin t\,dt$ **3**

OR

(b) (1) Using Laplace Transform, solve the differential equation **4**

$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^t$ with y (0) = 2, y′(0) = -1.

(2) Find : $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$. **3**

Q.3 **(a)** Solve the following differential equations : **6**

(1) $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-x}+\sin 2x.$

(2) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+2x+4.$

(b) Using method of variation of parameters, solve the differential equation: $\left(D^2+4\right)y=\tan 2x$. **4**

(c) The differential equation for a circuit in which self-inductance and capacitance neutralize each other is $L\frac{d^2i}{dt^2}+\frac{i}{C}=0$. Find the current i as a function of t given that I is the maximum current and $i=0$ when $t=0$. **4**

OR

Q.3 **(a)** Solve the following differential equations : **6**

(1) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=e^{-2x}\sin 2x$

(2) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=4\cos^2 x.$

(b) Solve : $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x).$ **4**

(c) The deflection of a strut of length l with one end (x = 0) built-in and the other supported and subjected to end thrust P satisfies the equation $\frac{d^2y}{dx^2}+a^2y=\frac{a^2R}{P}(l-x)$. Prove that the deflection curve is $y=\frac{R}{P}\left(\frac{\sin ax}{a}-l\cos ax+l-x\right)$, where $al = \tan al$. **4**

Q.4 **(a)** Find the Fourier series to represent $x-x^2$ from $x=-\pi$ to $x=\pi$. **5**

(b) Find the Fourier series in the interval (-2,2) if $f(x)=\begin{cases}0; -2<x<0\\ 1; 0<x<2.\end{cases}$ **5**

(c) Find the Fourier cosine transform of $f(x)=e^{-ax}$; Hence evaluate $\int\frac{\cos\lambda x}{x^2+a^2}dx.$ **4**

OR

Q.4 **(a)** Find the Fourier series to represent the function *f(x)* given by **5**

$f(x)=\begin{cases}x; for 0\le x\le\pi\\ 2\pi-x; for \pi\le x\le 2\pi.\end{cases}$

(b) Find the Fourier series to represent πx in the interval $0\le x\le 2$. **5**

(c) Using the Fourier sine transform of $e^{-ax}(a>0)$, show that $\int\frac{x\sin kx}{a^2+x^2}dx=\frac{\pi}{2}e^{-ak}(k>0).$ **4**

Q.5 **(a)** Attempt the following: **6**

(1) Solve : $x(y-z)p+y(z-x)q=z(x-y).$

(2) Solve : $x^2p^2+y^2q^2=z^2.$

(b) Attempt the following : **4**

(1) Form the partial differential equation from $z = f\left(\frac{xy}{z}\right)$.

(2) Solve : $pq+p+q=0$.

(c) Attempt the following : **4**

(1) Find z-transform of $a^k \cos \alpha k; k \geq 0$.

(2) Find the inverse z-transform of $\frac{z}{(z-2)(z-3)}; |z| > 3$.

OR

Q.5 **(a)** Attempt the following : **6**

(1) Solve : $(y+z)p-(z+x)q = x-y$.

(2) Solve : $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

(b) Solve by the method of separation of variable $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$. **4**

(c) Attempt the following : **4**

(1) Find the z-transform of $ka^k; k \geq 0$.

(2) Find the inverse z-transform of $\frac{z^2}{(z-1)(z-\frac{1}{2})}; |z| > 1$.

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