

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC SEM-II Examination-Dec-2011

Subject code: X20001**Date: 20/12/2011****Subject Name: Mathematics -II****Time: 10.30 am -1.30 pm****Total marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) (i) Prove that $\sqrt{m+1} = m\sqrt{m}$ **02**
(ii) $L(e^{4t} t^2) =$ **02**
03
(iii) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan \theta} d\theta = \frac{\pi}{\sqrt{2}}$
(b) (i) Show that $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$ **02**
(ii) $L(t \sin t) =$ **02**
03
(iii) Prove that $\int_0^1 \left(\log \frac{1}{y} \right)^{n-1} dy = \sqrt[n]{n}$, $n > 0$
- Q.2** (a) (i) Evaluate $\int_0^{\infty} e^{-3t} \sin 4t dt$ using Laplace transform. **03**
(ii) State convolution theorem. Find $L^{-1} \left(\frac{1}{(s-2)(s-1)} \right)$ **04**
(b) (i) Define unit step function. Find Laplace of unit step function. **03**
(ii) Using Laplace transform solve $y'' - y' - 6y = e^{-t}$, $y(0) = y'(0) = 0$ **04**
- OR**
- (b) (i) Evaluate: $L(e^{5t} t^2 \sin t)$ **03**
(ii) Evaluate: $L \left(e^{3t} \int_0^t \frac{\sin t}{t} dt \right)$ **04**
- Q.3** (a) (i) Draw the graph of periodic function $f(x) = 1$ if $0 < x < 2$ and $f(x) = -1$ if $-2 < x < 0$. Check whether $f(x)$ is even or odd. **03**
(ii) Find the Fourier series for $f(x) = x^2$, $-\pi < x < \pi$, $f(x + 2\pi) = f(x)$. **04**
(b) (i) Write Dirichlet's Conditions for Fourier series. **03**
Fourier series of $\tan^{-1}(x)$ in interval $(0, 2\pi)$ does not exist. (true/false).
(ii) Find Fourier integral representation of the function **04**
 $f(x) = 1$ if $|x| < 1$, $f(x) = 0$ if $|x| > 1$.

OR

- Q.3 (a)** (i) If $f(x) = e^{|x|}$, $-\pi < x < \pi$, $f(x + 2\pi) = f(x)$ then check $f(x)$ is even or odd also find Fourier coefficient b_n . **03**
- (ii) Find the Fourier series for $f(x) = -k$ if $-\pi < x < 0$ and $f(x) = k$ if $0 < x < \pi$, $f(x + 2\pi) = f(x)$ **04**
- (b)** (i) Find $F_c(e^{-x})$. **03**
- (ii) Expand $\pi x - x^2$ in a half range sine series in interval $(0, \pi)$. **04**
- Q.4 (a)** (i) Solve $(D^2 - D - 12)y = e^{2x} + 5$ **03**
- (ii) Solve by method of variation of parameter $D^2y + y = \sec x$ **04**
- (b)** (i) Solve $(D^2 - 2D + 1)y = \sin x$ **03**
- (ii) Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2$ **04**
- OR**
- Q.4 (a)** In L-C-R circuit the charge q on a plate of condenser is given by **07**
- $$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin \omega t$$
- The circuit is tuned to resonance so that $\omega^2 = 1/LC$. If $R^2 < 4L/C$ and $q = 0 = \frac{dq}{dt}$ when $t = 0$. Find $q(t)$
- (b)** (i) Solve $(1 + 2x)^2 \frac{d^2y}{dx^2} - 6(1 + 2x) \frac{dy}{dx} + 16y = 8(1 + 2x)^2$ **03**
- (ii) If in a mass spring system mass = 4kg spring constant = 64, $f(t) = 8 \sin 4t$ and if there is no air resistance then find the subsequent motion of the weight. **04**
- Q.5 (a)** (i) Form partial differential equation from following equation. (A) $z = ax + by + ab$ (B) $z = f(x^2 - y^2)$. **03**
- (ii) (A) Solve $y^2zp + x^2zq = xy^2$ **04**
- (B) Solve $(y - z)p + (x - y)q = z - x$
- (b)** (i) (A) Solve $z = px + qy + p^2q^2$ (B) Solve $p^2 + q^2 = 2$. **03**
- (ii) Solve difference equation $U_{k+1} + U_k = 1$ if $U_0 = 0$ **04**
- OR**
- Q.5 (a)** (i) Prove that Z-transform is linear. **03**
- (ii) Solve the following equation by method of separation of variables $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ where $u(0, y) = 8e^{-3y}$. **04**
- (b)** A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each its points a velocity $\lambda x(L - x)$, find the displacement of the string at any distance x from one end at any time t . **07**
