Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - Ist Semester–Examination – May/June- 2012

Subject code: X10001

Subject Name: Mathematics-I

Date:29/05/2012 **Time: 10:30 am – 01:30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$ **03**

(ii) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ **04**

(b) (i)If $\theta = t^n e^{-r^2/4t}$,what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$? **04**

(ii) Trace the curve $y^2(2a - x) = x^3$ **03**

Q.2 **(a)** (i)Using Gauss Jordan method, find the inverse of the matrix **03**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(ii) Find the Eigen values and Eigen vectors of the matrix **04**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(b) Solve the following differential equations:

(i) $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - x/y\right)dy = 0.$ **04**

(ii) $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0.$ **03**

OR

(b) Solve the following differential equations:

(i) $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$ **04**

(ii) $x\frac{dy}{dx} + y\log y = xy\, e^x$ **03**

Q.3 Attempt the following:

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$,show that **05**

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}$$

(b) If $u = \sin^{-1}\left[\frac{x + y}{\sqrt{x} + \sqrt{y}}\right]$,prove that **05**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4\cos^3 u}$$

(c) If $x = r\cos\theta$ and $y = r\sin\theta$; evaluate $\frac{\partial(x, y)}{\partial(r, \theta)}$ & $\frac{\partial(r, \theta)}{\partial(x, y)}$ **04**

OR

Q.3 Attempt the following:

(a) If $u = f(r)$, where $r^2 = x^2 + y^2$, **05**

prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

(b) If $z = x\, f\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$; **05**

show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$

(c) Examine the function $f(x, y) = x^3 + y^3 - 3axy$ for maxima & minima. **04**

Q.4 Attempt the following:

(a) Evaluate $\iint_R y\, dx\, dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. **05**

(b) Change the order of integration & evaluate $\int_0^a \int_y^a \frac{x\, dx\, dy}{x^2 + y^2}$ **05**

(c) Using double integration, find area lying between the parabola $y = 4x - x^2$ and the line $y = x$. **04**

OR

Q.4 Attempt the following:

(a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by changing to polar coordinates. **05**

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\, dy\, dz}{\sqrt{(1 - x^2 - y^2 - z^2)}}$ **05**

(c) Calculate the volume of the solid bounded by the planes $x = 0,\ y = 0,\ x + y + z = 1$ and $z = 0$. **04**

Q.5 Attempt the following:

(a) A particle moves along the curve $x = t^3 + 1,\ y = t^2,\ z = 2t + 3$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$. **05**

(b) Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = grad\,(x^3 + y^3 + z^3 - 3xyz)$ at the point $(1, 2, 3)$ **05**

(c) The rate at which body cools is proportional to the difference between the temperature of the body and that of the surrounding air. If body in air at $25^\circ C$ will cool from 100° *to* 75° in one minute, find its temperature at the end of three minutes. **04**

OR

Q.5 Attempt the following:

(a) Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point $A(1, -1, -1)$ in the direction of the line AB where B has coordinates $(3, 2, 1)$ **05**

(b) Use Green's theorem to evaluate $\int_c (x^2 + xy) dx + (x^2 + y^2) dy$ where C is the square formed by the lines $y = \pm 1,\ x = \pm 1$. **05**

(c) Find the orthogonal trajectories of the family of curves $x^2 - y^2 = c$. **04**

****