

GUJARAT TECHNOLOGICAL UNIVERSITY**PDDC - SEMESTER-I • EXAMINATION – SUMMER 2013****Subject Code: X10001****Date: 03-06-2013****Subject Name: Mathematics - I****Time: 02.30 pm - 05.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1

- (a) Reduce the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$ into row echelon form and find its rank. **05**
- (b) Solve following linear system by Gauss – Jordan elimination method. **05**
- $$3x + 3y + 2z = 1$$
- $$x + 2y = 4$$
- $$10y + 3z = -2$$
- $$2x - 3y - z = 5$$
- (c) Solve $(x^2 - yx^2) \frac{dy}{dx} + (y^2 + xy^2) = 0$. **04**

Q.2

- (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **07**
- (b) 1) Solve $(x^2 - y^2)dx - xydy = 0$. **04**
- 2) Solve $(x^2 - ay)dx = (ax - y^2)dy$ **03**

OR

- (b) Trace the curve $y^2(a + x) = x^2(a - x)$ **07**

Q.3

- (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}$ **05**
- (b) If $x^2 = au + bv$ and $y^2 = au - bv$ then show that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2}$. **05**
- (c) Show that $\text{grad}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$. **04**

OR

- (a) Find the maximum and minimum value of the function $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. **07**
- (b) Evaluate the line integral $\int_C [(5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}]$ where C is the curve $y = x^3$ from the point (1,1) to (2,8). **07**

Q.4

- (a) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$ prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin 2u$. **07**
- (b) 1) Find curl $\vec{F}$ at the point (1,2,3), where $\vec{F} = x^2 yz \vec{i} + xy^2 z \vec{j} + xyz^2 \vec{k}$. **04**
- 2) Show that $\vec{F} = (-x^2 + yz)\vec{i} + (4y - z^2 x)\vec{j} + (2xz - 4z)\vec{k}$ is solenoidal. **03**

OR

- Q.4** (a) If $x = r \cos \theta$, $y = r \sin \theta$, prove that $JJ' = 1$. **07**
 (b) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$, where C is bounded by $y = x$ and $y = x^2$. **07**

- Q.5** (a) Change the order of integration in $I = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$ and hence evaluate. **05**
 (b) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$. **05**
 (c) Find the orthogonal trajectories of the family of the parabolas $y = ax^2$. **04**

OR

- Q.5** (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dy dx$ by changing to polar coordinates. **05**
 (b) Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) dx dy dz$. **05**
 (c) Find the directional derivatives of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. **04**
