Seat No.: _____ Enrolment No. ______

GUJARAT TECHNOLOGICAL UNIVERSITY

P.D.D.C Semester – II (Civil/Mech/Elect/E.c)

Subject Name: MATHEMATICS - II

Subject code:X20001 **Total Marks: 70**

Date: **Time:3 Hours**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define Gamma function. Prove that $\Gamma n+1 = n!$, n is + ve integer. **02**

(b) Define Beta function. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **03**

(c) Prove that $\beta(m,n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$. **03**

(d) Find the Fourier cosine transform of f(x) = e^{-x^2}. **03**

(e) Express the function f(x) = $\begin{cases} \frac{\pi}{2}, 0 < x < \pi \\ 0, x > \pi \end{cases}$ **03**

As Fourier sine integral and show that $\int_0^{\infty} \frac{1-\cos\pi\omega}{\omega} \sin\omega x\, d\omega = \frac{\pi}{2}, 0 < x < \pi$.

Q.2 **(a)** Evaluate: (i) L($\cos^2 2t$) (ii) L($t^2 \sin 2t$) **04**

(b) State and prove I^{st} shifting property of Laplace transformation. **03**

(c) Attempt following:

(i) Evaluate: (a) $L^{-1}\left(\frac{(s^2-1)^2}{s^5}\right)$ (b) $L^{-1}\left(\frac{s+2}{s^2-4s+13}\right)$ **04**

(ii) State convolution theorem. Using it evaluate $L^{-1}\left(\frac{1}{(s+a)(s+b)}\right)$ **03**

OR

(c) Attempt following:

(i) Evaluate: (a) $L^{-1}\left(\frac{s+7}{s^2+2s+2}\right)$ (b) $L^{-1}\left(\cot^{-1}\left(\frac{s}{a}\right)\right)$ **04**

(ii) Using Laplace transformation solve the differential equation: $y'' - y = t, y(0) = y'(0) = 1$. **03**

Q.3 **(a)** To solve following differential equation: **05**

(i) $(D^2 + 2D + 1)y = \cos^2 x$

(ii) $(D^2 - 6D + 13)y = 8e^{3x} \sin 4x$

(b) Using method of variation of parameter to solve the differential equation: $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$. **05**

(c) Solve the differential equation: **04**

$x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + y = \log x$.

OR

Q.3 **(a)** To solve the following differential equation: **05**

(i) $\frac{d^2 y}{dx^2} - 4y = (1+e^x)^2 + 3$.

(ii) $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$.

(b) Using method of variation of parameters to solve the differential equation: $\frac{d^2 y}{dx^2} + y = \cos ecx$ **05**

(c) Solve the differential equation: **04**

$(1+x)^2 \frac{d^2 y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$.

Q.4 **(a)** Obtain Fourier series for f(x) = x (2π-x) in interval $0 < x < 2\pi$. **05**

(b) Obtain Fourier series for f(x) = $\begin{cases} \pi x, 0 \le x \le 1. \\ \pi(2-x), 1 \le x \le 2 \end{cases}$ **05**

(c) Obtain half-range sine series for f(x) = $\begin{cases} x, 0 \le x \le \frac{\pi}{2} \\ \pi - x, \frac{\pi}{2} \le x \le \pi \end{cases}$ **04**

OR

Q.4 **(a)** Obtain Fourier series for f(x) = x + x^2 in $-\pi < x < \pi$. **05**

Hence deduce that $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$.

(b) Obtain Fourier series for f(x) x – x ^{2} ,-1 < x <1. **05**

(c) Obtain Half – range cosine series for the function **04**

f(x)=$(x-1)^2$ in0<x<1.Hencededuce

that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7} + \ldots = \frac{\pi^2}{8}$.

Q.5 **(a)** Form the partial differential equation of **05**

(i) $z = (x^2 + a)(x^2 + b)$ (ii) $z = x + y + f(xy)$

(b) Solve the following partial differential equation **05**

(i) xp+yq = z (ii) ptanx + qtany = tanz

(c) Find Z-transform of (i) λ^n (ii) coshx **04**

OR

Q.5 **(a)** Solve following partial differential equation by direct integral method. **05**

(i) $\frac{\partial^2 z}{\partial x \partial y} = \cos x \cos y$ (ii) $\frac{\partial^2 z}{\partial y^2} = z$ given that when y = 0, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$.

(b) Solve the following partial differential equation: **05**

(i) $z = p^2 + q^2$ (ii) $p^2 + q^2 = x + y$.

(c) State and prove linear property for Z-transform. **04**

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