

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: X 10001**Date: 11/01/2013****Subject Name: Mathematics - I****Time: 10.30 am - 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (i) Define the rank of Matrix. Determine the rank of Matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ **03**

(ii) Is the Matrix $\begin{bmatrix} 1 & -5 & 4 & 5 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ in row echelon form or reduced row echelon form? **02**

(iii) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$. **02**

(b) (i) Solve the following system of equations:
 $x + y + z = 6, x + 2y + 3z = 4, x + 4y + 9z = 6$. by Gauss-elimination and back substitution method. **04**

(ii) Using Gauss-Jordan method find the inverse of the matrix **03**

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}.$$

Q.2 (a) (i) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **04**

(ii) State the Euler's theorem for homogeneous function and if **03**

$$u = \sin^{-1} \left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}} \right) \text{ then prove that } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{20} \tan u.$$

(b) (i) If $y = f(x + 2t) + g(x - 2t)$, prove that $\frac{\partial^2 y}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}$. **04**

(ii) If $x = r \cos \theta, y = r \sin \theta$ then find $\frac{\partial(x, y)}{\partial(r, \theta)}$. **03**

OR

(b) (i) Discuss the maxima and minima of $x^2 + y^2 + 6x + 12$. **04**

(ii) Find the equation of tangent plane at the point (1,1,1) on the surface $x^2 + y^2 + z^2 = 3$. **03**

Q.3 (a) Solve the following differential equations:

(i) $(x^2 - y^2)dx = (2xy)dy$. **04**

03

(ii) $\frac{dx}{dy} + x = y$.

- (b) (i) Solve the exact differential equation 04
 $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$.
(ii) Find the orthogonal trajectories of the family of the curve $x^2 - y^2 = c$. 03

OR

- Q.3** (a) Solve the following differential equations: 04
(i) $\frac{dy}{dx} - \frac{y}{x+1} = e^x(x+1)$, 03
(ii) $(e^y + 1)\cos x dx + e^y \sin x dy = 0$. 04
(b) (i) Solve $\cos(x+y)dy = dx$. 03
(ii) Find the orthogonal trajectories of the family of the curve $r^2 = c \sin(2\theta)$. 10

- Q.4** (a) Trace the curves: 04
(i) $r = a(1 + \cos \theta)$, $a > 0$. (ii) $xy^2 = 9a^2(2a - x)$.
(b) Evaluate $\int_0^2 \int_0^2 (x^2 + y^2) dx dy$. 04

OR

- Q.4** (a) Trace the curves: 04
(i) $y^2(a - x) = x^3$, (ii) $r^2 = a^2 \cos 2\theta$.
(b) Change the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$ and evaluate it. 03

- Q.5** (a) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$ by changing into polar coordinates. 04
(ii) Evaluate $\int_0^a \int_0^{a-x} \int_0^{a-x-y} x dz dy dx$. 03
(b) (i) Find the constants a, b, c so that vector $(x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (-x + cy + 2z)\hat{k}$ is irrotational. 03
(ii) Obtain the area of an Ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, by using Green's theorem. 04

OR

- Q.5** (a) (i) Change the order of integration $\int_0^{\frac{1}{\sqrt{2}}} \int_x^{\sqrt{1-x^2}} y^2 dA$ and evaluate it. 04
(ii) Evaluate $\int_0^2 \int_0^x \int_0^{\sqrt{x+y}} z dx dy dz$. 03
(b) (i) Is the vector $\vec{v} = (x - 3y)\hat{i} + (y - 2z)\hat{j} + (x - 3z)\hat{k}$ solenoidal? 03
(ii) Find $\int_c \vec{F} d\vec{r}$, where $\vec{F} = \frac{y\hat{i} - x\hat{j}}{x^2 + y^2}$ and c is the circle $x^2 + y^2 = 4$ traversed counter clockwise. 04
