

GUJARAT TECHNOLOGICAL UNIVERSITY
PDDC - SEMESTER – II • EXAMINATION – WINTER 2012

Subject code: X 20001**Date: 12/01/2013****Subject Name: Mathematics - II****Time: 10.30 am - 01.30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Attempt the following.

a) Define Beta function. Evaluate $\int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}} dx$. 3

b) Evaluate $\int_0^{\infty} e^{-x^2} dx$ in terms of gamma function. 3

c) Define Gamma function and evaluate $\Gamma 4.5$ 2

d) Find the Laplace Transform of $\sin 2t \sin 3t$. 2

e) Solve : $(D^2 + 6D + 9)y = 0$ 2

f) Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ 2

Q.2 (a) Find the inverse Laplace Transforms of 6

1) $\frac{3s+2}{s^2-s-2}$

1) $\cot^{-1}\left(\frac{s}{2}\right)$

(b) Find the Laplace Transformations of 4

1) $e^{-3t}(2 \cos 5t - 3 \sin 5t)$

2) $\frac{1-e^t}{t}$

(c) Using Laplace Transform solve $y'' + y = t, y(0) = 1, y'(0) = 0$. 4**Q.3** (a) Solve the following differential equations: 6

1) $(D^2 - 3D + 2)y = \cos 3x$

2) $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 4y = e^x \cos x$

(b) Using method of variation of parameters, solve the differential equation : 4

$y'' - 6y' + 9y = e^{3x} / x^2$

(c) Find half range sine series of $f(x) = x$ in $0 < x < 2$ 4**Q.4**(a) Find the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the range 0 to 2π . 5

- (b) Find the Fourier series for the function $f(x) = \begin{cases} \pi x & \text{if } 0 \leq x \leq 1 \\ \pi(2-x) & \text{if } 1 \leq x \leq 2 \end{cases}$ 5
- (c) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$ 4
- Q.5** (a) Find the Fourier series for the function $f(x) = x - x^2$ in the interval $-\pi \leq x \leq \pi$. 5
- (b) Solve $(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin[\log(1+x)]$. 5
- (c) Find the Z-transforms of 4
- 1) e^{an}
 - 2) $(n+1)^2$
- Q. 6** (a) Solve the following equations: 6
- 1) $p \tan x + q \tan y = \tan z$
 - 2) $(z-y)p + (x-z)q = y-x$
- (b) Form the partial differential equation from 4
- 1) $(x-a)^2 + (y-b)^2 + z^2 = c^2$.
 - 2) $z = f(x^2 - y^2)$
- (c) Using the Fourier integral representation, show that 4
- $$\int_0^{\infty} \frac{\omega \sin x \omega}{1 + \omega^2} d\omega = \frac{\pi}{2} e^{-x} \quad (x > 0).$$
- Q. 7** (a) Obtain the complete solution of the equations: 6
- 1) $p(1+q) = qz$
 - 2) $p+q = \sin x + \sin y$
- (b) Find the Fourier sine transform of $e^{-|x|}$. 4
- (c) Solve the equation $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, given that $u(0, y) = 8e^{-3y}$ by the method of separation of variable. 4
