Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

PDDC - SEMESTER–I • EXAMINATION – WINTER 2013

Subject Code: X10001 **Date: 17-12-2013**

Subject Name: Mathematics - I

Time: 10.30 am - 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Each question carry equal marks (14 marks)**

Q.1 Do as directed.

(a) Using Gauss-Jordan method to find the inverse of a matrix $A=\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ **4**

(b) Find the rank of matrix A by row echelone form, where **4**

$A=\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

(c) Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$ **4**

(d) What is the order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} \Big/ \frac{d^2y}{dx^2} = c$ **2**

Q.2 (a) If $u = \tan^{-1}\frac{x^3+y^3}{x^2+y^2}$, prove that **5**

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$

(b) Solve $\begin{aligned} 4x - 2y + 6z &= 8 \\ x + y - 3z &= -1 \\ 15x - 3y + 9z &= 21 \end{aligned}$ by Gauss elimination method. **5**

(c) Solve : $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ **4**

Q.3

(a) Find the eigen values eigen vectors of the matrix $A=\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ **5**

(b) Fid the maximum and minimum values of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ **5**

(c) Solve: $x\frac{dy}{dx} + y = x^3y^6$ **4**

Q.4 (a) Trace the curve $y^2(a-x) = x^2(a+x)$ **5**

(b) The period of a simple pendulum is $T = 2\pi\sqrt{l/g}$, find the maximum error in T due to the possible error upto1% in l and 2.5% in g. **5**

(c) If $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ **4**

Q.5 (a) Trace the curve $r^2 = a^2 \cos 2\theta$. **5**

(b) Evaluate $div\overline{F}$ and $curl\overline{F}$ at the point $(1,2,3)$, where **5**
$\overline{F} = grad(x^3y + y^3z + z^3x - x^2y^2z^2)$

(c) Evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)dxdy$ by changing the order of integration. **4**

Q. 6 (a) Find directional derivative of $\phi = x^4 + y^4 + z^4$ at the point $P(1,-2,1)$ in the direction PQ where Q is $(-1,6,-1)$. **4**

(b) Using Green's theorem, evaluate $\int_C [(xy + y^2)dx + x^2 dy]$ where C is bounded by $y = x$ and $y = x^2$. **4**

(c) Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2)dxdydz$ **3**

(d) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)dydx$ by changing to polar co-ordinates. **3**

Q. 7 (a) If $\overline{F} = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$, evaluate $\int_C \overline{F} \cdot d\bar{r}$ along the curve C in the XY- plane, $y = x^3$ from the point $(1,1)$ to $(2,8)$. **4**

(b) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. **4**

(c) Find the value of 'a' if the vector **3**
$\overline{F} = (ax^2y + yz)\bar{i} + (xy^2 - xz^2)\bar{j} + (2xy - 2x^2y^2)\bar{k}$ has zero divergence.

(d) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. **3**

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