

GUJARAT TECHNOLOGICAL UNIVERSITY**PDDC - SEMESTER-I • EXAMINATION – WINTER • 2014****Subject Code: X10001****Date: 30-12-2014****Subject Name: Mathematics-I****Time: 10:30 am - 01:30 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Find the Inverse of the Matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by Gauss-Jordan Method. **07**

(b) (I) Find the rank of Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ by determinant method. **03**

(II) Solve the following system of linear equations by Gauss-Elimination method : $4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21$. **04**

Q.2 (a) Trace the curve: $y^2(a^2 + x^2) = x^2(a^2 - x^2)$. **07**

(b) Find the Eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ **07**

OR

(b) Find the maximum and minimum value of the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ **07**

Q.3 (a) State Euler's theorem and if $u = \tan^{-1}(x^2 + 2y^2)$,
prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u$ **07**

(b) (I) If $x = r \cos \theta, y = r \sin \theta$ & $z = z$ then find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$. **03**

(II) If $x = r \cos \theta, y = r \sin \theta$ and $r = u^2 + v^2, \theta = 2uv$ then find $\frac{\partial(x, y)}{\partial(u, v)}$. **04**

OR

Q.3 (a) Change the order of integration and evaluate $\int_0^1 \int_{4y}^4 e^{x^2} dA$. **07**

(b) (I) Solve: $y' + y \sin x = e^{\cos x}$. **03**

(II) Solve: $y \sin 2x dx - (1 + y^2 + \cos^2 x) dy = 0$ **04**

Q.4 (a) (I) Evaluate: $\int_0^\pi \int_0^{1-\sin \theta} r^2 \cos \theta dr d\theta$ **03**

(II) Evaluate: $\int_0^1 \int_0^\pi \int_0^\pi y \sin z dx dy dz$ **04**

(b) Find the area of the region bounded by the curves $y = x^2$ & $y = x + 2$. **07**

OR

Q.4 (a) Verify Green's theorem for $\oint_C [(x - y)dx + 3xydy]$, where C is boundary of the region bounded by parabolas $x^2 = 4y$ & $y^2 = 4x$. **07**

(b) (I) Find the unit vector normal to the surface $x^2y + 2xz^2 = 8$ at the point (1,0,2). **03**

(II) Find the directional derivative of the function $\phi = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2$ & $z = t^3$ at the point (1,1,1). **04**

Q.5 (a) Solve: $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$ **07**

(b) Trace the curve $r = a(1 + \cos \theta), a > 0$. **07**

OR

Q.5 (a) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoid and irrotational. **07**

(b) Find the orthogonal trajectories of the family of the circles $x^2 + y^2 = 2cx$. **07**
